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BUBBLE INTERACTION IN LOW-VISCOSITY LIQUIDS

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Abstract—An experimental study investigated how freely rising ellipsoidal bubbles approach each other, make contact and coalesce or breakup. Pulsed planar swarms of 10–20 bubbles with Eötvös numbers from 6.0 to 27.5 were released simultaneously in aqueous solutions of 0–48 wt% sugar with Morton numbers from 3.2×10^{-11} to 3.7×10^{-6}. Bubble interaction was recorded by a video camera following the rising bubbles. Essentially, all coalescence and breakup events occurred after, not during, wake-induced collisions by a complex process related to the bubble vortex shedding cycle. This same process was also found in multi-bubble clusters and may account for excess turbulent kinetic energy generation in bubbly flow.

Key Words: bubbles, coalescence, breakup, bubble swarms, turbulence, bubbly flow

1. INTRODUCTION

The mechanisms by which freely rising bubbles behave in relatively low-viscosity liquids and, specifically, how they approach, contact and coalesce or break up is an important aspect of multiphase flow. Coalescence and breakup can control the interfacial area and mass transfer rate in bubble columns and gas-sparged chemical and biological reactors. Bubble interaction is fundamental in two-phase flow instability that plagues boilers and oil and gas wells. But the actual mechanics of bubble interaction remains relatively mysterious.

Models that predict the dynamics of bubble swarms, including coalescence and breakup rates, interfacial area transport and bubble size distributions, must be based on real physical phenomena. Bubbly flow instability, for example, has typically been treated as a kinematic wave, described mathematically by the eigenvalues of a linearized system of mass and momentum equations for the gas and liquid fields. But the mathematics of the stability conditions so derived could be much more clearly interpreted if the dynamics of the bubble interaction creating the kinematic wave were known.

This paper presents the results of a study of bubble interaction that provides new insights into the process. An experiment using pulsed swarms of relatively few bubbles revealed a basic interaction process that appears to be fundamental to bubble coalescence and breakup. It may also account for bubble flow instability and the transition from bubbly to slug flow.

Neglecting the density and viscosity of the gas phase, the shape and dynamic behavior of a single bubble rising in a quiescent, homogeneous liquid can be predicted with three independent dimensionless groups. These are commonly defined as the Reynolds number, Eötvös number and Morton number. These are given, respectively, by

$$\mathrm{Re} = \frac{\rho U_b D_e}{\mu}$$

and

$$\mathrm{E\ddot{o}} = \frac{g\rho D_e^2}{\sigma}$$

and

$$\mathrm{M} = \frac{g\mu^4}{\rho\sigma^3}$$

where the density and viscosity are those of the liquid. The equivalent diameter, D_e, is given in terms of bubble volume, V_b, by

$$D_e = \left[\frac{6}{\pi} V_b \right]^{1/3}$$

Bubble shape can be correlated with some precision on a map of Re versus Eö with M as a parameter (Grace *et al.* 1976). Differences in bubble behavior and shape transitions in liquids of differing viscosities led to the classification of liquids as "high-Morton number" or "low-Morton number". The terminal rise speed of bubbles in the latter increases with size to a local maximum just past the size where they become ellipsoidal, while the speed of bubbles in "high-M" liquids increases monotonically. Bhaga & Weber (1981) established the boundary between low-M and high-M liquids at $M = 4 \times 10^{-3}$ where the peak in terminal velocity begins to appear.

A bubble's dynamic behavior is intimately related to its shape. Ellipsoidal bubbles in low-M liquids exhibit an unsteady wobbling rise path. Wobbling begins at a Reynolds number of about 200. The wobble period is constant once established, but its onset depends on Morton number (Haberman & Morton 1953; Hartunian & Sears 1957; Tsuge & Hibino 1977). From photographs of the wake, Lindt & DeGroot (1974) found that the wobble period became constant when what they described as an attached helical vortex reached its maximum length. They further observed that the transition from ellipsoidal to cap shape was coupled with significant changes in wake structure. The turbulent wake consists of a chain of looping horseshoe vortices originating around the periphery of the bubble's base (Yabe & Kunii 1978).

The bubble wake is the main driver for interaction. If a bubble enters the rising column of liquid in another's wake under the right conditions, the two bubbles can make contact and coalesce. Wake-induced collisions result in coalescence primarily between pairs of large cap bubbles in fluids sufficiently viscous to keep their wakes laminar. But pairs of smaller bubbles or large bubbles in low-M fluids do not. Small spherical or ellipsoidal bubbles tend to repel each other except under very specific conditions. The turbulent wake behind bubbles in less-viscous liquids has a weaker downstream influence than a laminar one because it is intermittent and irregular. Turbulence in the wake often causes trailing bubbles to break up spontaneously or upon collision with the leader. Collisions in low-viscosity liquids occur at high relative velocity which prevents coalescence by trapping a liquid barrier between them (Nevers & Wu 1971; Crabtree & Bridgwater 1971; Narayanan *et al.* 1974; Kirkpatrick & Lockett 1974; Komasawa *et al.* 1980; Bhaga & Weber 1980; DeKee *et al.* 1986; Kumaran & Koch 1993).

Individual coalescence and breakup events are essentially impossible to observe in a swarm. Indirect, non-visual measurements of bubble size and chemical concentration can be used to infer interaction intensity and coalescence and breakup rates. But these tests are very difficult to interpret in terms of actual bubble behavior. Instrumentation has not yet been able to resolve the details of the coalescence event. However, consistent with the behavior of the in-line pairs, viscous liquids enhance coalescence in bubble swarms and achieve larger bubbles. Turbulence and periodic vortex shedding in the less viscous liquids is more apt to cause breakup than coalescence. The transition from "coalescing" to "non-coalescing" is approximately at the $M > 4 \times 10^{-4}$ value observed for the transition between "low-M" and "high-M" liquids (Calderbank *et al.* 1964; Otake *et al.* 1977; Oolman & Blanch 1976; Greaves & Barigou 1992).

Beyond this, the bubble interaction process in swarms is highly complex and not well understood. This motivated an experimental study that attempted to simplify the system to make the details visible while containing enough bubbles to represent a true swarm.

2. EXPERIMENT DESIGN

Our objective was to discover how bubble interaction occurs in swarms of freely rising bubbles in low-viscosity liquids. In order to observe enough bubbles to represent a real swarm, but not so many that the mechanisms are hidden, a *pulsed planar swarm* of 10–20 bubbles was released simultaneously into a test section 20 × 20-cm square, and 2 m high. For a complete description of the apparatus and conditions see Stewart (1993) and Hiller (1993). Aqueous solutions of sugar in

Table 1. Properties of aqueous sugar solutions at 17 C

Mass fraction	0.0	0.168	0.274	0.365	0.440	0.481
Specific gravity	1.0000	1.065	1.111	1.154	1.192	1.214
Density (kg/m^3)	999	1064	1110	1153	1191	1213
Viscosity ($N\text{-}s/m^2$)	0.0011	0.0020	0.0028	0.0046	0.0089	0.0189
Surface tension (N/m)	0.0738	0.064	0.064	0.062	0.064	0.065
Morton number	3×10^{-11}	5×10^{-10}	2×10^{-9}	2×10^{-8}	2×10^{-7}	4×10^{-6}

demineralized water were used with Morton numbers ranging from 3.2×10^{-11} to 3.73×10^{-6} in steps of approximately one order of magnitude. Great care was taken to prevent contamination of the water with surfactants or electrolytes and periodic analysis confirmed the purity of the solution. The properties of these sugar solutions are shown in table 1.

A rotating plate release mechanism provided up to 16 bubbles in a staggered square array. This is shown in plan view in figure 1. A set of plates with 12 larger release positions corresponding to the inner three rings of figure 1 was also used. Each bubble release position was filled separately with a measured volume of air introduced into a section of clear, small-diameter tubing outside the test section under the same pressure head as the release plate. The volume was metered by adjusting the bubble length in the tube to a calibrated scale. The swarm was released by rotating the fill plate in the horizontal plane with a stepper motor to align each fill position with matching exit holes in a cover plate above it. A side view of the fill and release operation is sketched in figure 2.

Bubble interactions were recorded by an 8 mm video camera that traversed to follow the rising swarm. The video tapes were later studied in detail by viewing each run frame-by-frame and at reduced speeds (1/10 to 1/5 speed). A total of 4963 bubbles were released in 343 runs recorded on more than 3.5 h of video tape. From the video data, 1672 binary wake-induced collisions were recorded of which only 108 resulted in a breakup and 56 in coalescence.

The maximum bubble size ranged from 0.65 to 1.28 cm equivalent diameter, with Eötvös numbers from 6 to 28. They assumed an irregular wobbling spiral path consistent with results of past studies (Tsuge & Hibino 1977; Lindt & DeGroot 1974). In all solutions with $M < 2 \times 10^{-7}$ the wobble period was uniformly 0.2 ± 0.03 s. In the most viscous sugar solution, $M = 4 \times 10^{-6}$ and Re = 100–130, bubbles did not wobble except after a collision or under the influence of another's wake. At

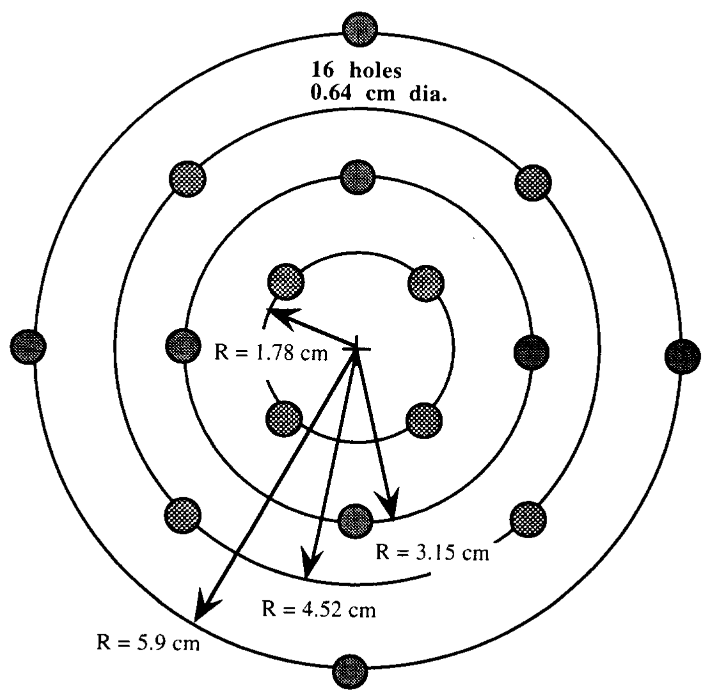

Figure 1. Bubble release plate configuration.

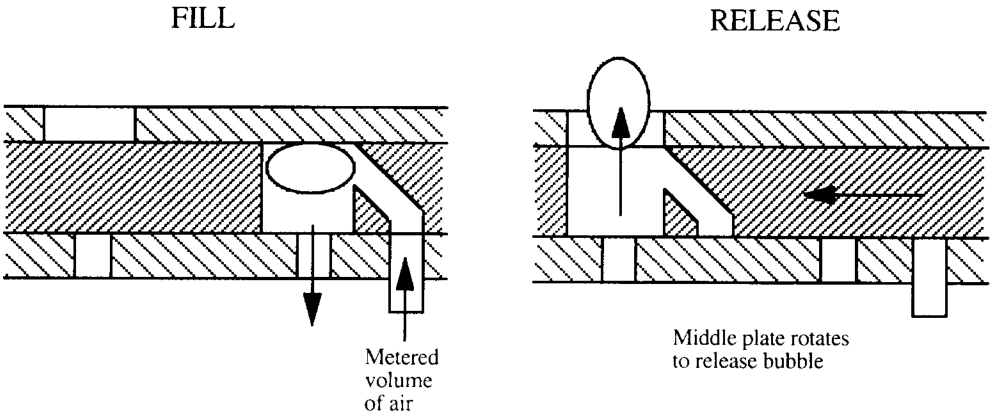

Figure 2. Bubble fill and release process.

$M = 2 \times 10^{-7}$ and Re = 300–400, all bubbles wobbled but at a smaller amplitude than those at lower M.

The wobble cycle was not a pure spiral, but more of a zig–zag with a complex flexing and stretching motion superimposed. Figure 3 shows a 1.25 cm (Eö = 20) bubble in water through one full wobble cycle. The cycle runs from bottom to top, each frame 1/30 s apart. The complex, continually evolving shape may be evidence of periodic discharges of excess vorticity down the wake.

3. BINARY BUBBLE INTERACTION

The wake was observed to be the driving force and sole mechanism for bubble interaction. No head-on or lateral solid-body collisions were observed. No bubble approached another except by entering its wake and overtaking it from the rear. Wake capture normally led to a violent "rear-ender" collision ending with the two bubbles nearly side-by-side about one diameter apart. The bubbles usually stopped interacting at this point. No coalescences or breakups were ever observed in the collision itself. An example of a complete collision event is shown in figure 4.

The first indication that a bubble was entering another's wake was a disturbance or pause in its wobble pattern at about 6 equivalent diameters behind the leader. Once captured by the wake the trailing bubble began overtaking the leader immediately, though erratically. Trailing bubbles occasionally left the wake after traveling a few diameters in it.

Instead of accelerating smoothly as in high-M liquids, the trailing bubble appeared to overtake the leader in a series of jumps. During a jump, the bubble appears to either elongate considerably or tilt to rise edgewise. The actual collision occurred after a final jump of 2–4 equivalent diameters. This was the longest and fastest jump in what was evidently the strongest and most coherent part of the leader's wake. It was often difficult to resolve the details of this complex process. Actual contact lasted only a fraction of one wobble period, about 1/10 s. The overtaking bubble appeared to be drawn up into the center of the leader's rear face, sometimes disappearing completely inside it. However, it very quickly pushed past and slightly ahead; the two bubbles separated slightly, and re-established their own wobble cycles.

The collision was (with rare exceptions) the only path to either coalescence or breakup, but neither was ever observed during the actual collision process. Unless the bubbles assumed a "key" position after collision, with one slightly ahead of the other, exposing its near wake to the rear one, the bubbles separated with no further interaction. No bubbles achieved the key position without first colliding.

Once in the key position, the bubbles "danced" together, sometimes for several wobble cycles, before coalescing, breaking up or drifting apart again. One cycle of the "dance" is sketched in figure 5. When the leader's periphery dips downward, it draws the trailer down and inward under its edge, as in frame three of the figure. In the next frame, the leader rocks back away from the dip, stretching the neighbor further around in a U-shape toward it. If the bubble stretches enough to overcome surface tension, breakup, coalescence or both may occur independently. Otherwise, the lower bubble recovers its shape and position for the next wobble as frames five and six indicate. A dramatic example of a simultaneous breakup and coalescence is shown in figure 6.

Coalescences were always binary and occurred preferentially between bubbles of approximately similar size. In fact, no coalescences were observed between bubbles of widely disparate sizes. In many cases, the entire coalescence process occurred in as little as one video frame (1/30 s) and seldom in more than three frames (about 1/10 s). The actual interface penetration appeared to be instantaneous. Most of the time involved the two volumes flowing together into one. This is quite different from

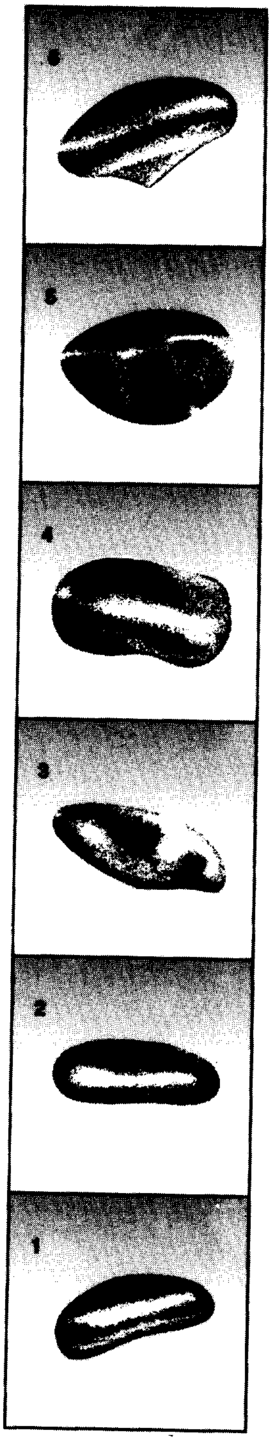

Figure 3. Single bubble wobble cycle.

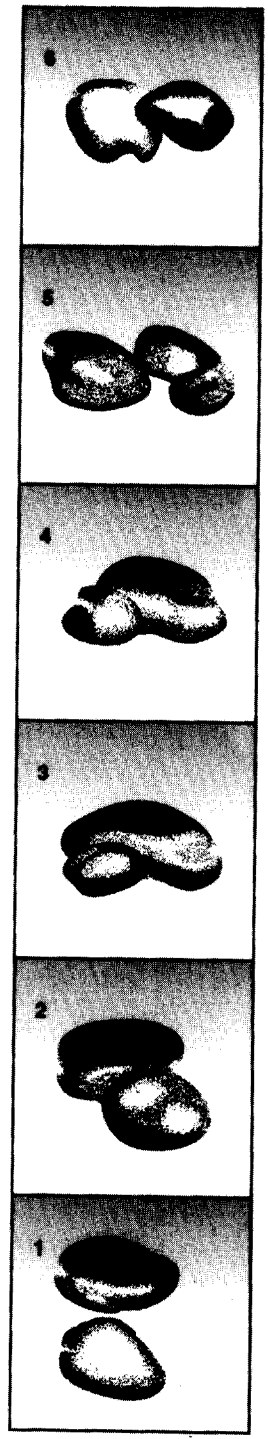

Figure 4. Typical collision event.

the accepted model requiring bubbles to stay in contact until the intervening liquid film between the two bubbles drains away.

Breakup was always binary, preferentially spawning non-uniform daughter sizes. While no measurement was possible, the daughter sizes were visually consistent with the data of Hesketh *et al.* (1991) for breakup in horizontal pipe flow. Their photographs indicate that the breakups might also be the result of a downstream bubble's wake. Those shown by Walter & Blanch (1986) of breakups in a swarm clearly appear to occur by this near-wake stretching process.

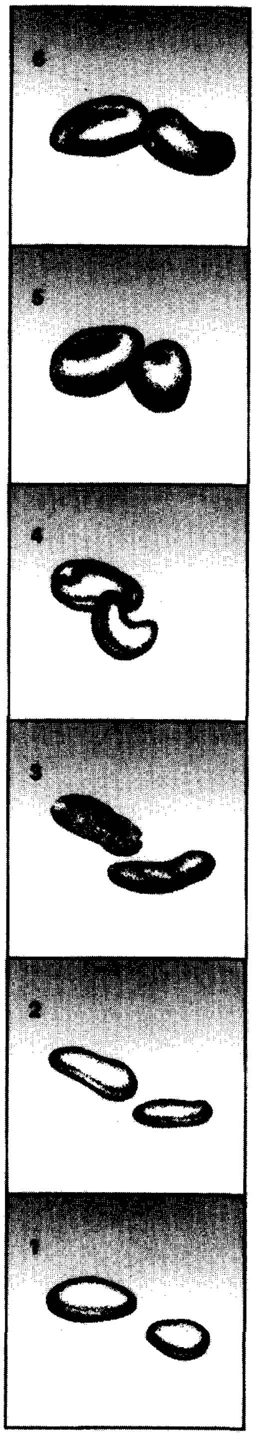

Figure 5. Post-collision "dance" interaction.

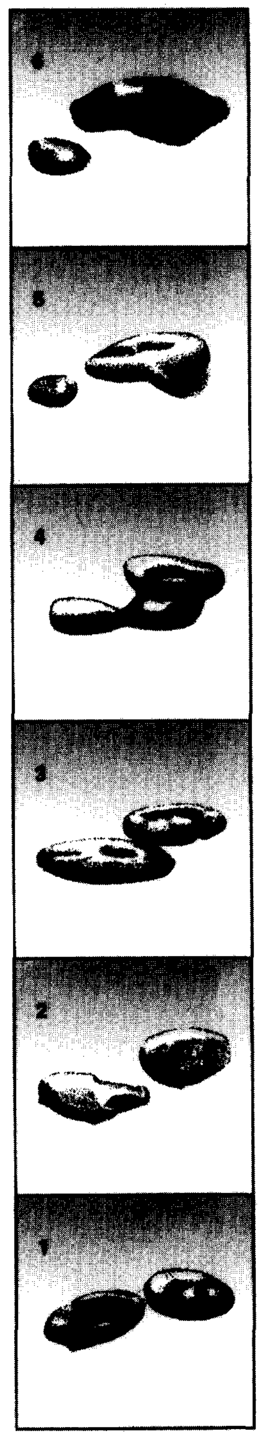

Figure 6. Simultaneous breakup and coalescence.

The observations that coalescence or breakup occurred only after wake collision with the bubble pair in a diagonally adjacent position is contrary to earlier observations where large bubbles in viscous liquids coalesced only during the wake collision, and then only with perfect vertical alignment. It appears that there is a transition related to the Morton number where the in-line collision coalescence mechanism ceases and the diagonal "dance" mechanism begins.

The transition may be in the range $10^{-6} < M < 10^{-4}$. Mao & Core (1993) observed that most coalescences occurred at the collision and required perfect vertical alignment with a Morton number near 10^{-4}, just over one order of magnitude higher than our maximum M of 4×10^{-6}. But they also did tests with the pair released simultaneously at the same height, 0.5 cm apart. These interacted and even made contact, but they only coalesced when one bubble was drawn into the center of the other. This is exactly the "dance" coalescence mechanism we observed. Otake *et al.* (1977) also saw bubbles coalesce on collision when the leading bubble overlapped more than half the trailing one with a $M{\sim}10^{-6}$. These experiments appear to bracket the two types of behavior.

4. MULTIPLE BUBBLE INTERACTION

When several bubbles were captured in the wake of another simultaneously or in rapid sequence, they formed a cluster. The dynamics of bubble clusters may be the basic foundation for the transition from bubbly to slug flow and for the kinematic waves or "void waves" observed in flowing two-phase systems of low-viscosity fluids.

The bubbles in a cluster did not act as a coherent unit as the word "cluster" implies, but continually traded places in a "leapfrog" fashion. When two bubbles climbed the wake of another together, or a third bubble collided with a pair that had just collided, one of the overtaking bubbles usually moved past the collision point ahead of the others. The new leader then captured one or both of the bubbles now just behind it and a second collision occurred. This sequence usually repeated several times before one of the bubbles broke the cycle and left the wake after a collision. Two cycles of typical cluster action are sketched in figure 7. Every other video frame is shown; each is 1/15 s apart.

The lifetime, rise speed, size and general violence of the cluster increased with the number of bubbles available to participate. If a large number of bubbles (say 5–10) were involved, or the cluster wake captured additional bubbles below, collisions became almost continuous and the cluster ceased to pause between advances. This formed a chimney in which a cluster's wake was strong enough to sustain itself by continually gathering in new bubbles to replace those that dispersed outward at the top. Figure 8 attempts to show the growth and dispersion of a chimney.

The dynamics of a chimney varied with the number and spacing of the individual bubbles in it. A chimney died out when bubbles dispersed at the top too widely to be re-captured or too few nearby bubbles were captured as it rose past them. As a chimney grew stronger, more breakups occurred and the smaller bubbles could not sustain the process. On the other hand, coalescences encouraged by the close proximity of bubbles in the cluster created large caps that greatly amplified the chimney's wake. But very large caps sometimes broke up spontaneously, nullifying much of the amplifying effect. Occasionally a second cluster formed in a chimney's wake and collided with the one at the head of the chimney. However, this just as likely dispersed as strengthened the chimney.

The wake capture/collision/separation process transfers mechanical energy from the bubbles to the liquid much more rapidly than if the individual bubbles rise independently. Assuming the wake created by a bubble moves with it at terminal rise speed, U_∞, an equal sized bubble captured by the wake will immediately accelerate to terminal velocity in the wake, making its absolute speed $2U_\infty$, gaining on the leader at a speed of U_∞. Thus the overtaking bubble is expending potential energy at twice the rate it would when rising independently.

It is easy to show that 1.5 times more energy is dissipated by the pair from wake capture to collision than if no interaction had occurred. Even recognizing that the wake decays with distance such that the effective bubble speed in the wake is reduced to CU_∞ where $1 < C < 2$, the energy dissipation per collision is still a factor of $(1 + C)/2$ above that of bubbles traveling the same distance independently. The potential energy thus dissipated appears as increased turbulent kinetic energy. If each collision represents an increment in turbulence, the high collision rates observed in large clusters and chimneys imply a large amplification of turbulent energy production.

Lance & Battaille (1991) found exactly this effect in their experiment that imposed grid-generated turbulence on a vertical co-current air–water flow of ellipsoidal bubbles at volume fractions under 3%. At low-volume fractions, they found the total turbulent kinetic energy was simply the sum of that generated by the grid and by the bubbles. But above a volume fraction threshold of about 0.01, the bubbles generated turbulent kinetic energy far in excess of the simple superposition. At the same time, the one-dimensional energy spectra shifted from the classic $-5/3$ law to a $-8/3$ dependence at the higher wave numbers. Lance & Battaille suggested that the bubble wake generates a great deal of energy at short length scales which dissipates before spectral transfer can occur.

5. CONCLUSIONS

The qualitative behavior of bubbles interacting in pulsed swarms of ellipsoidal bubbles in low-viscosity sugar solutions has been described based on the video tape record of the experiment. These observations have revealed some fundamental patterns that cannot be seen in dense continuous swarms and cannot occur with only a single bubble or a pair of bubbles. The findings are summarized as follows:

- Individual bubbles rise in an irregular but periodic spiral wobble with a period of about 0.2 s. The wobble pattern is evidently driven by vortex shedding in the near wake and is an integral part of all bubble interaction.
- A bubble contacts another only by following its wake to an overtaking collision. No interaction occurs except involving the wake. No coalescence or breakup occurs during the actual collision.

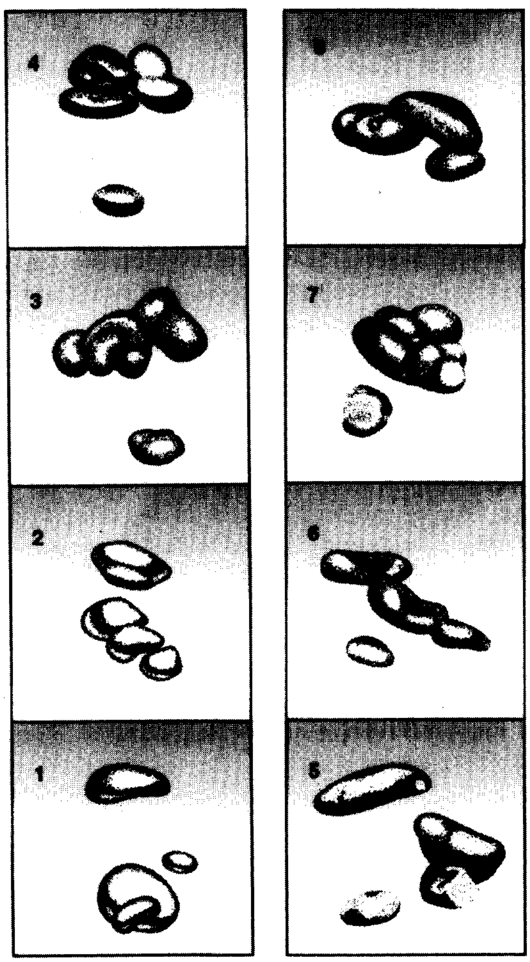

Figure 7. Cluster dynamics.

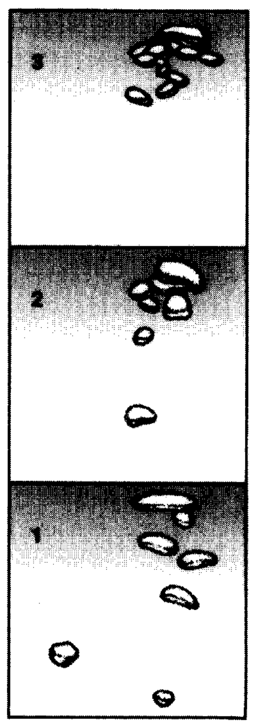

Figure 8. Bubble dynamics in a chimney.

- Essentially all coalescence and breakup events occur after the wake capture collision, when one bubble is pulled into the near wake of the other during the "dance" process. Breakup and coalescence may occur simultaneously.
- Breakup is always binary and tends to create non-uniform fragments. Coalescence is also binary and requires bubbles of approximately similar size.
- Interaction of three or more bubbles in clusters and "chimneys" may be the basic dynamics of flow regime transitions and excess energy dissipation in bubbly two-phase flow.

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